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Nonlinear Sigma Model in the Faddeev–Jackiw Quantization Formalism

A. Foussats,[1,2] C. Repetto,[1,2] O. P. Zandron,[1,2] and O. S. Zandron[1,2]

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The key equations of the symplectic Faddeev–Jackiw formalism are written in an alternative way so that the inverse of the symplectic matrix is easily found. The nonlinear sigma model including the Hopf term in the action is treated in the framework of this quantization method. It is shown how the complete dynamics of the system is described by means of the generalized Faddeev–Jackiw quantum brackets.

1. INTRODUCTION

From the theoretical point of view, as well as phenomenologically, anyons are important for several reasons. Anyonic excitation can be described by means of different theoretical approaches; see, for instance, Berezin and Marinov (1977), Leinass and Myrheim (1977), Goldin *et al.* (1980, 1981), Wilczek (1982; for a recent review see Wilczek, 1991), Wilczek and Zee (1983), Laughlin (1983), the articles in Chern *et al.* (1991), Wu and Zee (1984), Bowick *et al.* (1986), Dzyaloshinskii *et al.* (1988), Polyakov (1988), Plyushchay (1992), Hagen (1984, 1985), Arovas *et al.* (1985), the review by Jackiw (1990), Kogan (1991), Kogan and Semenoff (1992), Stern (1991), Cabo *et al.* (1992), Cortes *et al.* (1992, 1994), Chou *et al.* (1993), Chaichian *et al.* (1993), Frölich and Marchetti (1988), Hlousek and Spector (1990), and Foussats *et al.* (1996a, b). In this paper we do not consider these aspects because anyons and their properties have already been extensively discussed in the literature.

[1]Facultad de Ciencias Exactas Ingenieriá y Agrimensura de la UNR, 2000 Rosario, Argentina.
[2]Miembros del Consejo Nacional de Investigaciones Científicas y Técnicas, Argentina.

Among the different approaches, an interesting proposal is to study supersymmetric anyon theories in terms of the coupling of a conserved current superfield to a nondynamical gauge superfield with Chern–Simons term. In this framework it can be seen how the anyon–anyon interaction is determined only by supersymmetry requirements. In other words, in a supersymmetric theory, anyon species must interact in such a way that supersymmetry is preserved.

It is also known that in $(2 + 1)$ dimensions the topological solitons of the bosonic $O(3)$ nonlinear sigma model can be turned into anyons by including the Hopf term in the action. At the classical level, the supersymmetric generalization of this approach was developed in Hlousek and Spector (1990). For instance, following this approach, it is shown how apparent paradoxes appearing in supersymmetric anyon theories are solved by analyzing symmetry breaking, both supersymmetry and gauge symmetry breaking.

In the framework of the usual Dirac formalism for constrained Hamiltonian systems (Dirac, 1950, 1964), the study of the constraint structure of the bosonic $O(3)$ nonlinear sigma model with Hopf term requires long algebraic manipulations.

The purpose of the present work is to analyze this gauge model in the symplectic Faddeev–Jackiw quantum picture, leaving the study of the supersymmetric generalization of the model for a forthcoming paper.

An alternative way to treat constrained systems was proposed by Faddeev and Jackiw (FJ) (1988), which in some cases is more economical than Dirac's method. This happens because in the FJ construction there is in general a minor number of constraints.

The FJ symplectic formalism has been carefully studied (Costa and Girotti, 1988; Govaerts, 1990; Barcelos-Neto and Srivastava, 1991; Kulshreshtha and Muller-Kirsten, 1991; Barcelos-Neto and Wotzasek, 1992a, b, 1993; Horta-Barreira and Wotzasek, 1992; Montani and Wotzasek, 1993a,b), and a supersymmetric extension of this method to include Grassmann dynamical variables can be found in Govaerts (1990), but has not often been used in supersymmetric systems.

Recently, we have written the key equation of the supersymmetric extension of the symplectic FJ formalism in an alternative way (Foussats and Zandron, 1997). This allowed us to obtain general equations from which the generalized FJ commutators can be easily computed.

In Section 2 we briefly recall the main equations of the symplectic quantization formalism that are used in the quantization of the constrained nonlinear sigma model with Hopf current. In Section 3, we summarize the principal characteristics of the model, and then explicity evaluate the generalized FJ commutators.

2. AN ALTERNATIVE WAY OF WRITING THE KEY EQUATIONS IN THE SYMPLECTIC FADDEEV–JACKIW FORMALISM

In order to compute the generalized commutators, one of the crucial points in the FJ method is to study how difficult is the invertibility of the symplectic matrix.

In this section the main results of the symplectic formalism are reviewed by writing the key equations in a simple language that easily allows us to compute the inverse of the symplectic matrix.

The FJ symplectic quantization method is based on first-order Lagrangians. This is not a serious restriction because any system can be written in a first-order formalism by enlarging the configuration space by introducing proper auxiliary fields. As can be shown, the generalized brackets obtained from the equation of motion are equal to those obtained by means of the Dirac formalism, producing the same dynamical results. In the FJ symplectic formalism the classification of constrained or unconstrained systems is related to the singular or nonsingular behavior of the fundamental symplectic two-form. The classification of constraints as primary, secondary, and so on, or as first-class or second-class constraints has no meaning. Once the symplectic algorithm is finished, the only remaining constraints are those associated with gauge symmetries. So, in this method there is in general a minor number of constraints compared with the number of constraints generated by the Dirac algorithm. Hence, we can expect that the algebraic manipulations needed in the treatment of the constrained systems could be shortened.

The most general action containing first-order time derivatives is defined by a Lagrangian density written in terms of two arbitrary functionals $K_A(\varphi^A)$ and $\mathbf{V}(\varphi^A)$,

$$L(\varphi_A, \dot{\varphi}^A) = \dot{\varphi}^A K_A(\varphi^A) - \mathbf{V}(\varphi^A) \tag{2.1}$$

The functionals $K_A(\varphi^A)$ are components of the canonical one-form $K(\varphi) = K_A(\varphi)d\varphi^A$, and the functional $\mathbf{V}(\varphi)$ is the symplectic potential. The general compound index A runs in the different ranges of the complete set of variables. The set of field dynamical variables φ^A is given by the original set of fields plus a set of auxiliary fields necessary to bring the system into its first-order form (2.1) and this set defines the extended configuration space.

The Euler–Lagrange equations of motion obtained from (2.1) are

$$\sum_B M_{AB}\dot{\varphi}^B - \frac{\partial \mathbf{V}}{\partial \varphi^A} = 0 \tag{2.2}$$

The elements of the symplectic matrix $M_{AB}(\varphi)$ are components of the sympletic two-form $M(\varphi) = dK(\varphi)$. The exterior derivative of the canonical one-form $K(\varphi)$ is written as the generalized curl constructed with functional derivatives and so the components are given by

$$M_{AB}(x, y) = \frac{\delta K_B(y)}{\delta\varphi^A(x)} - \frac{\delta K_A(x)}{\delta\varphi^B(y)} \tag{2.3}$$

When the symplectic matrix M_{AB} is nonsingular, it defines the symplectic two-form characterizing the dynamical system described by (2.1). From the equations of motion (2.2) we obtain

$$\dot{\varphi}^A = (M^{AB})^{-1} \frac{\partial \mathbf{V}}{\partial\varphi^B} \tag{2.4}$$

As the symplectic potential is just the Hamiltonian of the system, the equation (2.4) is written

$$\dot{\varphi}^A = [\varphi^A, \mathbf{V}] = [\varphi^A, \varphi^B] \frac{\partial \mathbf{V}}{\partial\varphi^B} \tag{2.5}$$

where $[\varphi^A, \varphi^B] = (M^{AB})^{-1}$ are the generalized brackets of the FJ symplectic formalism. The elements $(M^{AB})^{-1}$ of the inverse of the symplectic matrix M_{AB} correspond to the graded Dirac brackets of the theory. Transition to the quantum theory is realized as usual by replacing classical fields by quantum field operators acting on some Hilbert space. Therefore, in this case the FJ and the Dirac methods are equivalent.

On the other hand, in gauge-invariant field theories, besides the true dynamical degrees of freedom, there are also gauge degrees of freedom, and so first-class constraints exist and the matrix M_{AB} is singular. That is the case of Lagrangian densities describing gauge theories.

In the FJ formalism the constraints appear as algebraic relations and they are necessary to maintain the consistency of the field equations of motion. In such a case, there exist m ($m < n$) left (or right) zero-modes $\mathbf{v}_{(\alpha)}$ ($\alpha = 1, \ldots, m$) of the matrix M_{AB}, where each $\mathbf{v}_{(\alpha)}$ is a column vector with $n + m$ entries $v^A_{(\alpha)}$. So the zero-modes satisfy the equation

$$\sum_A v^A_{(\alpha)} M_{AB} = 0 \tag{2.6}$$

where the compound indices $A = (i, \alpha)$ and $B = (j, \beta)$ run in the ranges $i, j = 1, \ldots, n$ and $\alpha, \beta = 1, \ldots, m$.

Consequently, from the equations of motion (2.2) we can write

$$\Omega_{(\alpha)} = \int dx\, v^A_{(\alpha)}(x, t) \frac{\delta}{\delta\varphi^A(x, t)} \int dy\, \mathbf{V}(y, t) = 0 \tag{2.7}$$

The quantities $\Omega_{(\alpha)}$ are the constraints in the FJ symplectic formalism, and they are introduced in the Lagrangian by using suitable Lagrange multipliers:

$$L = \dot{\varphi}^i K_i(\varphi) - \Lambda^{(\alpha)}\Omega_{(\alpha)} - \mathbf{V}(\varphi) \tag{2.8}$$

In equation (2.8) we have assumed that $\varphi^i(x)$ represents any field belonging to the symplectic set. Therefore, the submatrix $\overline{M}_{ij}$ of the matrix (2.3) is nonsingular.

At this point one can run the symplectic algorithm once again, enlarging the configuration space by considering the set of variables $(\varphi^i, \xi^{(\alpha)})$. This is done by redefining the $\Lambda^{(\alpha)}$ variables as

$$\Lambda^{(\alpha)} = -\dot{\xi}^{(\alpha)} \tag{2.9}$$

Therefore, the first-iterated Lagrangian is written

$$L^{(1)} = \dot{\varphi}^i K_i(\varphi) + \dot{\xi}^{(\alpha)}\Omega_{(\alpha)} - \mathbf{V}^{(1)}(\varphi), \tag{2.10}$$

where $\mathbf{V}^{(1)}(\varphi) = \mathbf{V}(\varphi)|_{\Omega_{(\alpha)}=0}$.

In terms of the new set of dynamical variables, the symplectic matrix in compact notation is written

$$M_{AB}^{(1)} = \begin{pmatrix} \overline{M}_{ij} & \dfrac{\delta\Omega_{(\alpha)}}{\delta\varphi^j} \\ -\left(\dfrac{\delta\Omega_{(\alpha)}}{\delta\varphi^j}\right)^T & 0 \end{pmatrix} \tag{2.11}$$

where $\overline{M}_{ij}$ (submatrix of M_{AB}) represents the square nonsingular matrix constructed from the original symplectic set of field variables. The notation $\delta\Omega_{(\alpha)}/\delta\varphi^j$ represents a rectangular matrix.

This iterative procedure modifies the symplectic matrix until all the nonorthogonal zero-modes have been eliminated. That means that the algorithm must be repeated until no new constraint is generated. As we will see, for gauge-invariant theories, the algorithm is not able to generate an invertible symplectic matrix. Therefore, to obtain the generalized brackets, gauge-fixing conditions can be imposed.

Now, by writing the following general expression for the inverse of the symplectic matrix M_{AB}

$$(M^{AB})^{-1}(x, y) = \begin{pmatrix} A^{jk}(x, y) & B^{j\rho}(x, y) \\ C^{\beta\kappa}(x, y) & G^{\beta\rho}(x, y) \end{pmatrix} \tag{2.12}$$

it can be seen that a unique quantity $(M^{AB})^{-1}$ exists with the property

$$\int dz\, M_{AB}(x, z)(M^{BC})^{-1}(z, y) = \delta_A^C \delta(x, y) \tag{2.13a}$$

$$\int dz (M^{AB})^{-1}(x, z) M_{BC}(z, y) = \delta_C^A \delta(x, y) \tag{2.13b}$$

The equations are

$$\int dz\, \overline{M}_{ij}(x, z) A^{jk}(z, y) + \frac{\delta\Omega_\beta(z)}{\delta\varphi^i(x)} C^{\beta\kappa}(z, y) = \delta_i^k \delta(x - y) \tag{2.14a}$$

$$\int dz\, \overline{M}_{ij}(x, z) B^{j\rho}(z, y) + \frac{\delta\Omega_\beta(z)}{\delta\varphi^i(x)} G^{\beta\rho}(z, y) = 0 \tag{2.14b}$$

$$-\int dz \frac{\delta\Omega_\alpha(x)}{\delta\varphi^j(z)} A^{jk}(z, y) = 0 \tag{2.14c}$$

$$-\int dz \frac{\delta\Omega_\alpha(x)}{\delta\varphi^j(z)} B^{j\rho}(z, y) = \delta_\alpha^\rho\, \delta(x - y) \tag{2.14d}$$

After some algebra we find

$$\begin{aligned} B^{j\rho}(x, y) &= -C^{\rho j}(y, x) \\ &= -\int dz\, dw (\overline{M}^{jk})^{-1}(x, w) \frac{\delta\Omega_\beta(z)}{\delta\varphi^k(w)} G^{\beta\rho}(z, y) \end{aligned} \tag{2.15}$$

$$\begin{aligned} A^{ij}(x, y) &= (\overline{M}^{ij})^{-1}(x, y) \\ &\quad - \int dz\, dw \left(\int du (\overline{M}^{ik})^{-1}(x, u) \frac{\delta\Omega_\beta(z)}{\delta\varphi^k(u)} \right) \\ &\quad \times \left(\int dv (\overline{M}^{jl})^{-1}(z, v) \frac{\delta\Omega_\alpha(w)}{\delta\varphi^l(v)} \right) G^{\alpha\beta}(w, y) \end{aligned} \tag{2.16}$$

$$\int dz\, \Omega_{\alpha\beta}(x, z) G^{\beta\rho}(z, y) = \delta_\alpha^\rho \delta(x - y) \tag{2.17}$$

where

$$\Omega_{\alpha\beta}(x, z) = \int dy\, dv \frac{\delta\Omega_\alpha(x)}{\delta\varphi^j(y)} (\overline{M}^{ji})^{-1}(y, v) \frac{\delta\Omega_\beta(z)}{\delta\varphi^i(v)} \tag{2.18}$$

Finally, the generalized brackets are given by

$$\begin{aligned} [\varphi_i(x), \varphi_j(y)] &= (\overline{M}^{ij})^{-1}(x, y) \\ &\quad - \int dz\, dw \left(\int du\, (\overline{M}^{ik})^{-1}(x, u) \frac{\delta\Omega_\beta(z)}{\delta\varphi^k(u)} \right) \\ &\quad \times \left(\int dv\, (\overline{M}^{jl})^{-1}(z, v) \frac{\delta\Omega_\alpha(w)}{\delta\varphi^l(v)} \right) G^{\alpha\beta}(w, y) \end{aligned} \tag{2.19}$$

Equations (2.17)–(2.19) show that the generalized FJ brackets can be computed only if $\Omega_{\alpha\beta}$ and $\overline{M}^{ij}$ are invertible. Therefore, all algebraic manipulations are reduced to compute the matrices $(\overline{M}^{ij})^{-1}$ and $G^{\alpha\beta}$.

In the next section we apply the above results to the nonlinear sigma model in the presence of the topological Hopf current.

3. THE NONLINEAR SIGMA MODEL WITH HOPF TERM IN THE FADDEEV–JACKIW PICTURE

In (2 + 1) dimensions the topological solitons of the bosonic $O(3)$ nonlinear sigma model can be turned into anyons by including the Hopf current. This conspicuous feature renders the model interesting from the quantum point of view.

The bosonic $O(3)$ nonlinear sigma model including the topological Hopf current term, as a constrained system, gives rise to a singular symplectic matrix. As will be seen, a way to remove the singularity is to break the gauge symmetry in the symplectic potential by adding gauge-fixing terms.

The Lagrangian density for this model can be written in terms of a set of real scalar fields ϕ^a and a $U(1)$ gauge field A_μ as follows:

$$\mathcal{L}^0 = \frac{1}{2}\,\partial^\mu\phi^a\,\partial_\mu\phi^a - 2\pi\theta J^\mu A_\mu + \pi\theta\epsilon^{\mu\nu\rho}A_\mu\partial_\nu A_\rho + \lambda(\phi^2 - 1) \quad (3.1)$$

where the topological Hopf current is defined by

$$J^\mu = \frac{1}{8\pi}\,\epsilon^{\mu\nu\rho}\epsilon_{abc}\phi^a\partial_\nu\phi^b\partial_\rho\phi^c \quad (3.2)$$

and the non-linear constraint $\phi^2 = 1$ is introduced in (3.1) by using a Lagrange multiplier. The index $a = 1, 2, 3$ runs in the adjoint representation of $O(3)$ and from now on the Greek indices μ, ν, $\rho = 0, 1, 2$, and the Latin indices $i, j = 1, 2$.

From (3.1) it is easy to write the first-order Lagrangian by introducing as dynamical variable the canonical conjugate momentum π^a of the field ϕ_a,

$$\pi^a = \dot{\phi}^a + \frac{1}{2}\,\theta\epsilon^{abc}\epsilon^{ij}\phi_b\partial_i\phi_c A_j \quad (3.3)$$

In the case of dealing with the Hamiltonian Dirac method, the components of the canonical conjugate momentum P^μ of the gauge field A_μ also must be considered, and both components (P^0, P^i) are constraints. So, there are primary constraints on which Dirac consistency conditions must be imposed, increasing in this way the algebraic manipulations.

In the Lagrangian FJ formalism no new variables appear, because the Lagrangian density is already of first order in the $U(1)$ gauge field A_μ. The initial set of symplectic variables defining the extended configuration space is given by the set $(\phi_a, \pi^a, A_\mu, \lambda)$, and so the starting Lagrangian density is written in first-order form as follows:

$$\mathcal{L}^0 = \dot{\phi}_a \pi^a + \pi\theta\epsilon^{ij} A_j \dot{A}_i - V^0(\phi_a, \pi^a, A_\mu, \lambda) \tag{3.4}$$

where the symplectic potential is given by

$$V^0(\phi_a, \pi^a, A_\mu, \lambda) = \frac{1}{2}\pi_a\pi^a - \frac{1}{2}\partial_i\phi^a\partial^i\phi_a - \frac{1}{2}\theta\pi^a N^j_a A_j + \frac{1}{8}\theta^2 N^{aj} N^i_a A_j A_i + 2\pi\theta(J^0 - \epsilon^{ij}\partial_i A_j)A_0 - \lambda(\phi^2 - 1) \tag{3.5}$$

In equation (3.5), to simplify notation we have denoted

$$N^{aj} = \delta^{abc}\epsilon^{ij}\phi_b\partial_i\phi_c \tag{3.6}$$

Initially, in the case under consideration the 10×10 (singular) symplectic matrix (2.3) reads

$$M^0_{AB} = \begin{pmatrix} \overline{M}^0_{\Lambda\Sigma} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \tag{3.7}$$

The nonsingular 8×8 matrix $\overline{M}^0_{\Lambda\Sigma}$ is constructed from the original set of nonsingular field variables (ϕ_a, π^a, A_i), and is given by

$$\overline{M}^0_{\Lambda\Sigma} = \begin{pmatrix} 0 & -\delta_{ab} & 0 \\ \delta_{ab} & 0 & 0 \\ 0 & 0 & -2\pi\theta\epsilon^{ij} \end{pmatrix} \tag{3.8}$$

From the expression (3.7) it can be seen that there are two zero-modes. The corresponding zero-modes are given by

$$\mathbf{v}_1 = (\mathbf{0}_a, \mathbf{0}_a, \mathbf{0}_i, v_1, 0) \tag{3.9a}$$

$$\mathbf{v}_2 = (\mathbf{0}_a, \mathbf{0}_a, \mathbf{0}_i, 0, v_2) \tag{3.9b}$$

where v_1 and v_2 are arbitrary.

The constraints are evaluated from equation (2.7) and they read

$$\Omega_1 = 2\pi\theta(J^0 - \epsilon^{ij}\partial_i A_j) = 0 \tag{3.10a}$$

$$\Omega_2 = \phi^2 - 1 = 0 \tag{3.10b}$$

The constraint (3.10a) is precisely the time component of the field equation for the auxiliary nonphysical $U(1)$ field A_μ, and (3.10b) is none other than the nonlinear constraint of the sigma model.

Now we must carry out the first iterative procedure, and so the expression for the first-iterated Lagrangian is

$$\mathcal{L}^0 \to \mathcal{L}^1 = \dot{\phi}_a \pi^a + \pi\theta\epsilon^{ij} A_j \dot{A}_i + \dot{\xi}_1 \Omega_1 + \dot{\xi}_2 \Omega_2 - V^1 \tag{3.11}$$

where V^1 is defined by

$$V^1 = V^0|_{\Omega_1=\Omega_2=0} = \frac{1}{2}\pi_a \pi^a - \frac{1}{2}\partial^i \phi_a \partial_i \phi^a - \frac{1}{2}\theta \pi^a N^j_a A_j + \frac{1}{8}\theta^2 N^{ai} N^j_a A_i A_j \tag{3.12}$$

The modified symplectic matrix obtained after the first iteration is completed is again singular. As can be seen, there are two new zero-modes associated to this matrix and they are written in terms of two new arbitrary quantities v_3 and v_4:

$$\mathbf{v}_3 = (\mathbf{0}_a, -2\phi_a v_3, \mathbf{0}_i, 0, v_3) \tag{3.13a}$$

$$\mathbf{v}_4 = \left(\mathbf{0}_a, \frac{3}{4} M_a v_4, \mathbf{0}_i, v_4, 0\right) \tag{3.13b}$$

Using once more the equation (2.7), we find

$$\Omega_3 = 2\phi_a \pi^a = 0 \tag{3.14a}$$

$$\Omega_4 = \frac{3}{4}\theta M_a Z^a = 0 \tag{3.14b}$$

where

$$M^a = \epsilon^{abc}\epsilon^{ij}\partial_i \phi_b \partial_j \phi_c \tag{3.15}$$

$$Z^a = \pi^a - \frac{1}{2}\theta N^{ai} A_i \tag{3.16}$$

After some algebra we can show the following relation among constraints:

$$\phi^2 \Omega_4 + \frac{3}{4}\theta N^{aj} Z_a \partial_j \Omega_2 - 3\pi\theta J^0 \Omega_3 = 0 \tag{3.17}$$

that is, Ω_4 is not a new constraint. Moreover, in the soliton case $J^i \propto N^{ai} Z_a = 0$, and Ω_4 is proportional to Ω_3.

At this stage, the theory has three constraints and the matrix $\Omega_{\alpha\beta}(x, y)$ is obviously singular. A new iterative step is necessary and the Lagrangian is

$$\mathcal{L}^1 \to \mathcal{L}^2 = \dot{\phi}_a \pi^a + \pi\theta\epsilon^{ij} A_j \dot{A}_i + \dot{\xi}_\alpha \Omega_\alpha - V^2 \tag{3.18}$$

where now $\alpha = 1, 2, 3$; the three constraints Ω_α are given in equations (3.10a), (3.10b), and (3.14a), and V^2 is defined by the equation

$$V^2 = V^1|_{\Omega_\alpha=0} = V^1 \tag{3.19}$$

Repeating the above procedure, another one zero-mode appears that can be written in components

$$\mathbf{v}_5 = \left(\mathbf{0}_a, -\frac{3}{4}\,\theta\left[\frac{8\pi J^0}{\phi^2}\,\phi_a - M_a\right]v_5, -\frac{1}{2}\,\theta N_a^i\partial_i v_5, -\partial_i v_5, v_5, \frac{3}{8}\,\theta u_5, 0\right) \tag{3.20}$$

and has a constraint associated, which can be written in terms of the old constraints as follows:

$$\Omega_5 = \Omega_4 - 3\pi\theta\,\frac{J^0}{\phi^2}\,\Omega_3 \tag{3.21}$$

At this stage the procedure is finished because no new constraint is found. As corresponding to a gauge theory, the final symplectic matrix is singular.

As noted above, the invertibility of the matrix $\Omega_{\alpha\beta}$, and therefore the invertibility of the symplectic matrix are performed by breaking the symmetry in the symplectic potential.

This can be done by means of a gauge-fixing term added to the Lagrangian density. Such a term plays the role of a new constraint. The simplest case is to consider the gauge of divergenceless for the $U(1)$ gauge field A_μ.

Consequently, the set of constraints we must take into account is (3.10a), (3.10b), (3.14a), and the gauge-fixing term

$$\Omega_4 = \partial^i A_i = 0 \tag{3.22}$$

Now, by computing the matrix elements $\Omega_{\alpha\beta}(x, y)$ we obtain

$$\Omega_{13}(x, y) = \frac{1}{2}\,\theta\phi_a(x)(M^a(x) + 2N^{ai}(x)\partial_i)\delta(x - y) \tag{3.23a}$$

$$\Omega_{14}(x, y) = -\nabla^2\delta(x - y) \tag{3.23b}$$

$$\Omega_{23}(x, y) = -4\phi^2(x)\delta(x - y) \tag{3.23c}$$

and the other matrix elements are all zero.

The matrix $G_{\alpha\beta}$ defined in (2.17) necessary to compute the generalized FJ brackets is now easily found and has the form

$$G_{\alpha\beta}(x, y) = \begin{pmatrix} 0 & 0 & 0 & \frac{1}{\nabla^2} \\ 0 & 0 & \frac{1}{4\phi^2} & 3\pi\theta \frac{J^0}{\phi^2}\frac{1}{\nabla^2} \\ 0 & -\frac{1}{4\phi^2} & 0 & 0 \\ -\frac{1}{\nabla^2} & -3\pi\theta\frac{J^0}{\phi^2}\frac{1}{\nabla^2} & 0 & 0 \end{pmatrix} \delta(x - y) \tag{3.24}$$

Finally, the generalized FJ brackets (2.19) in the divergenceless gauge (3.22) have the following expressions:

$$[\phi_a(x), \phi_b(y)] = 0 \tag{3.25a}$$

$$[\phi_a(x), \pi_b(y)] = \left(\delta_{ab} - \frac{\phi_a\phi_b}{\phi^2}\right)\delta(x - y) \tag{3.25b}$$

$$[\pi_a(x), \pi_b(y)] = \frac{1}{\phi^2}(\pi_a\phi_b - \pi_b\phi_a)\delta(x - y) \tag{3.25c}$$

$$[\phi_a(x), A_j(y)] = 0 \tag{3.25d}$$

$$[\pi_a(x), A_j(y)] = -\frac{1}{4\pi}\epsilon_{jk}N_a^i(x)\partial_i^{(x)}\partial^{(y)k}G(x, y) \tag{3.25e}$$

$$[A_i(x), A_j(y)] = \frac{1}{2\pi\theta}[\epsilon_{ij}\delta(x - y) + (\epsilon_{ik}\partial_j^{(x)} - \epsilon_{jk}\partial_i^{(x)})\partial^{(y)k}G(x, y)] \tag{3.25f}$$

where the functional $G(x, y)$ defined above satisfies the differential equation

$$\nabla^2 G(x, y) = \delta(x - y) \tag{3.26}$$

The above generalized FJ brackets correspond to the Dirac brackets of the model. Therefore, once the nonsingular symplectic matrix is found, the complete canonical information about the dynamical system is obtained. The symplectic matrix also contains the complete information about all the symmetries present in the model.

As noted above, the transition to quantum theory is realized as usual in a canonical formalism by replacing classical fields by quantum field operators acting on some Hilbert space.

4. CONCLUSIONS

In summary, we have found the constraints in the bosonic $O(3)$ nonlinear sigma model including the topological Hopf current term.

In the framework of the FJ symplectic method the generalized commutators are found by removing the singularity in the symplectic matrix. This was done by adding the gauge-fixing term in the symplectic potential.

It is clear that the zero-modes of the symplectic matrix constructed by this method are closely related to the generators of gauge symmetries. In this context the unique constraints are those associated to gauge symmetries, and so the role of generators of the gauge symmetries assigned to these first-class constraints is clear.

The algebraic manipulations needed to find the constraints are less than with the Dirac procedure.

The supersymmetric extension of the nonsigma linear model with super Hopf topological current is an interesting model to explain anyon physics by means of supersymmetry. On the other hand, the supersymmetric extension of the FJ formalism has not frequently been used. Therefore, a useful exercise would be to apply the equations given in Foussats and Zandron (1997) to this supersymmetric model.

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